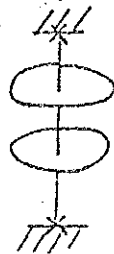


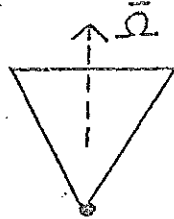
CLASSICAL MECHANICS

(Do All Problems)

1. A flatbed railroad car is rolling on a horizontal section of track. Assume there is no friction and therefore the speed is constant. Now consider what happens if a torrential rain storm occurs. Water falls on the railroad car and drips off onto the ground. Does this affect the speed of rolling of the car? Why? How does the speed depend on time? Assume there is no wind driving the rain, and that the vertical velocity of the rain drops is much greater than the horizontal velocity of the railroad car.
2. Consider a cube of homogeneous material with total mass M and linear dimension L . What is its moment of inertia for rotation about the body diagonal?
3. A piece of piano wire is stretched between two fixed supports. Two disks, each with the moment of inertia, I , are clamped onto the wire so that they divide it into three equal parts. This whole system can undergo torsional oscillation. Use the letter κ to denote the torsional Hooke's Law constant for each subsection of the wire.
 - a. Define a system of generalized coordinates and find the Lagrangian for the system.
 - b. Find the equations of motion.
 - c. Find the normal modes. Describe them, either with a sketch or in words.
 - d. Discuss what would happen to the normal modes if the central section of wire is made much longer or much thinner than the end sections.



- e. Discuss qualitatively what other normal modes exist for this system (i.e. other than the torsional ones).
4. A simple pendulum is made by attaching a bead with mass m to two equal length massless strings and fixing the other ends of the two strings to the ends of a supporting horizontal rod. In this way the pendulum is constrained to swing only in a plane which is perpendicular to the supporting rod. The rod is rotated about its center with angular frequency Ω .
- Express the Lagrangian of the bead in terms of a suitable generalized coordinate and find the equation of motion.
 - Find the equilibrium position of the mass as a function of Ω . Exercise some special care in discovering all physically meaningful solutions.
 - Find the frequency of small oscillations about the equilibrium as a function of Ω .
 - Sketch graphically the results of parts b and c.



5. Antiprotons can be produced by the reaction



Usually this is done by directing a beam of energetic protons onto a hydrogen target. In this situation what is the minimum kinetic energy of the beam protons for which this reaction can occur? When producing antiprotons with a beam having this threshold energy, what is the momentum of the antiprotons? And of one of the protons?

(Please express your answer in units in which the velocity of light c and the mass of the proton m_p both equal one.)

6. The oil-energy crisis has made popular discussions of many alternatives to current transportation systems. One such alternative is powering automobiles with energy stored in a flywheel. One of many difficulties with flywheels is that they act as gyroscopes and are difficult to turn when the auto turns or goes from level road onto a grade. How important is this problem? (Assume 300 KwHr or 10^9 joule of energy must be stored and the maximum angular frequency may reach 10^9 radians/sec).

Would such an auto have problems when parked because of the rotation of the earth? It has been suggested that by having two flywheels rotating in opposite directions one could cancel out the angular momentum and eliminate any problem. Would this work? What new problem might be introduced by such a tactic?

THERMODYNAMICS AND STATISTICAL MECHANICS

No textbooks or notes. Please do all problems.

1. An ideal gas has an isothermal change of volume from V_1 to V_2 at temperature T . Derive a formula for the heat absorbed.
2. A mass m of a liquid is at temperature T_1 . It is mixed with an equal mass of the same liquid at temperature T_2 . The system is thermally insulated, and the specific heat per gram is C_p . Calculate the change in entropy.
3. Derive a formula for the entropy of Black Body radiation in a volume V at temperature T in terms of V , T , and the fundamental constants h , c and k . Given that the energy density in a range of frequency ν is

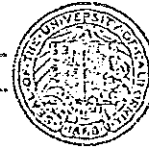
$$c^3 \int \frac{8\pi h \nu^3 d\nu}{e^{h\nu/kT} - 1}$$

4. The Fermi-Dirac distribution law for the probability f_k that a state with energy ϵ_k is occupied is

$$f_k = \frac{1}{e^{(\epsilon_k - \mu)/kT} + 1}$$

Suppose we have a Fermi gas composed of a very large number N of spin $\frac{1}{2}$ particles. Interactions between particles can be neglected and the largest kinetic energy at $T=0$ is ϵ_0 . Compute the average kinetic energy at $T=0$, in terms of ϵ_0 , assuming the system is homogeneous, isotropic and described by non relativistic mechanics.

5. A simple pendulum of mass m hangs by a string of length l . The string can be considered weightless. The pendulum hangs in a room having a gas at atmospheric pressure, and absolute temperature T . Let g be the acceleration due to gravity. Assume the pendulum is in thermal equilibrium as a result of bombardment by molecules of the gas, and its position relative to the vertical fluctuates. Compute the mean square value of the angle ϕ between the string and the vertical.



DEPARTMENT OF PHYSICS

IRVINE, CALIFORNIA 92664

Possibly Useful Information

$$\int_0^{\infty} e^{-a^2 x^2} dx = \frac{1}{2a} \sqrt{\pi}$$

$$\int_0^{\infty} x^{n-1} e^{-x} dx = \Gamma(n)$$

$$\int_0^{\infty} \frac{\sin^2 x dx}{x^2} = \frac{\pi}{2}$$

$$\int_0^{\infty} \frac{x^n dx}{e^x - 1} = \begin{array}{l} 1.645 \quad (n=1) \\ 2.405 \quad (n=2) \\ 6.49 \quad (n=3) = \pi^4/15 \\ 24.9 \quad (n=4) \end{array}$$

MATHEMATICAL PHYSICS

Do 4 problems. Use one open book.

1. A right-circular cylinder has fixed volume, radius r and height z . Find the relation between z and r which gives minimum total surface area for fixed volume.
2. A square stretched membrane ($0 \leq x \leq L$, $0 \leq y \leq L$) is subjected to a force $f(t)$ at the point $x = x_0$, $y = y_0$. There is a small damping proportional to the velocity of the membrane displacement. Find the general form for the membrane displacement $\psi(x, y, t)$. Express the result in terms of the Fourier transform of $f(t)$. Then, briefly consider the solution as the damping is removed.
3. (a) Using the identity

$$J_0(bt) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-ibt \sin x} dx$$

evaluate the integral

$$\int_0^{\infty} e^{-at} J_0(bt) dt$$

- (b) The integral

$$I = \int_0^{\infty} dx J_n^2(dx) \left[\frac{d}{dx} \left(a x e^{-a^2 x^2} \right) \right]$$

can in general be either positive or negative, depending on the constants d and a . Without doing the integral, find a qualitative condition between d and a which will insure that I is positive.

4. Consider the matrix equation

$$(L_0 + Q)u = \lambda u$$

where

$$L_0 = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}, Q = \begin{pmatrix} 0 & \epsilon & 0 \\ \epsilon & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and $\epsilon \ll 1$, i.e., Q is a perturbation.

(a) Find the zero-order eigenvalues, λ_0 , and the zero-order column vectors u_0 .

(b) Find the perturbed eigenvalues λ_1 .

5. Calculate the principal value of

$$\int_0^{\infty} \frac{\ln x}{x^{1/2}(1-x)} dx$$

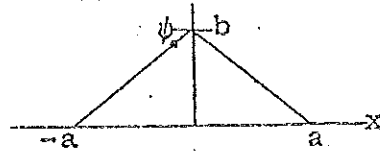
QUANTUM MECHANICS I

You may use one Quantum Mechanics textbook.

- 1) Consider a 1 dimensional harmonic oscillator

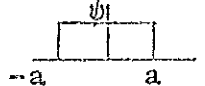
$$H = -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{k}{2} x^2$$

Minimize the energy for a triangular trial wave function



Compare your calculated value to the true ground state energy

for H. Discuss the use of a square trial wave function



- 2) Consider a particle of mass m scattered by a square well potential with a hard core

$$\begin{aligned} V(r) &= +\infty & r < c \\ &= -V_0 & c < r < a \\ &= 0 & r > a \end{aligned}$$

- a) Calculate the S wave phase shift δ as a function of energy.
- b) For the case $c = 0$ calculate δ in the Born approximation. Compare with your exact result from a) at high energy.
- 3) A deuteron (hydrogen 2 nucleus) consists of a neutron and proton in a (total) angular momentum state $J = 1$.
- a) Using the Clebsch-Gordan Coef in the table calculate all possible wave functions $\psi_{L,S}^{J=1, J_z=1}$ in terms of the intrinsic spin (S) wave functions χ for the n and p and an orbital angular momentum (L) wave function ϕ . (Do this neatly.)

b) Using

$$\mu_D = \langle \psi_{J=1, J_z=1} | \mu_z | \psi_{J=1, J_z=1} \rangle$$

$$\mu_z = (\underline{\mu})_z = (2\mu_n S_{n-z} + 2\mu_p S_{p-z} + \frac{1}{2} \underline{L})_z$$

with $\mu_n = -1.91$ (in units of nuclear Bohr magnetons)
and $\mu_p = 2.79$.

Calculate various possible magnetic moments μ_D of deuteron using the wave functions from part a).

c) Compare result with the experimental value μ_D of .86. Conclude that the deuteron is predominately an $S = 1$, $L = 0$ state. What other state can mix (via strong interactions) with it? What percentage mixture of this state would give the experimental μ_D ? How would you observe the presence of the other state experimentally?

TABLE VII
CLEBSCH-GORDAN COEFFICIENTS AND SPHERICAL HARMONICS

1/2x1/2				1x1/2				3/2x1/2			
m_1	m_2	M	M	m_1	m_2	M	M	m_1	m_2	M	M
1/2	1/2	1	1	1/2	1/2	1	1	3/2	1/2	2	2
1/2	-1/2			1/2	1/2			3/2	1/2		
-1/2	1/2			1/2	-1/2			3/2	-1/2		
-1/2	-1/2			1/2	-1/2	1	1	3/2	-1/2	1	1

2x1/2				3x1/2				5x1/2			
m_1	m_2	M	M	m_1	m_2	M	M	m_1	m_2	M	M
2	1/2	5/2	5/2	3	1/2	7/2	7/2	5	1/2	11/2	11/2
2	-1/2			3	1/2			5	1/2		
1	1/2			3	-1/2			5	-1/2		
1	-1/2			3	-1/2	1	1	5	-1/2	1	1

3x1				4x1				5x1			
m_1	m_2	M	M	m_1	m_2	M	M	m_1	m_2	M	M
3	1	4	4	4	1	5	5	5	1	6	6
3	0			4	1			5	1		
2	1			4	0			5	0		
2	0			4	0	1	1	5	0	1	1

3/2x3/2				4x3/2				5x3/2			
m_1	m_2	M	M	m_1	m_2	M	M	m_1	m_2	M	M
3/2	3/2	3	3	4	3/2	11/2	11/2	5	3/2	13/2	13/2
3/2	1/2			4	3/2			5	3/2		
3/2	-1/2			4	1/2			5	1/2		
1/2	3/2			4	1/2	1	1	5	1/2	1	1

3x3				4x3				5x3			
m_1	m_2	M	M	m_1	m_2	M	M	m_1	m_2	M	M
3	3	6	6	4	3	7	7	5	3	8	8
3	2			4	3			5	3		
3	1			4	2			5	2		
3	0			4	2	1	1	5	2	1	1

$$Y_0^0 = \sqrt{\frac{1}{4\pi}}$$

$$Y_1^0 = \sqrt{\frac{3}{4\pi}} \cos \theta; \quad Y_1^1 = \sqrt{\frac{3}{8\pi}} \sin \theta e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{16\pi}} \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right); \quad Y_2^1 = \sqrt{\frac{15}{32\pi}} \cos \theta \sin \theta e^{i\phi}$$

$$Y_2^2 = \frac{1}{2} \sqrt{\frac{15}{16\pi}} \sin^2 \theta e^{2i\phi}$$

$$Y_3^0 = \sqrt{\frac{7}{16\pi}} \left(\frac{7}{2} \cos^3 \theta - \frac{3}{2} \cos \theta \right); \quad Y_3^1 = \frac{1}{2} \sqrt{\frac{21}{16\pi}} \sin \theta (5 \cos^2 \theta - 1) e^{i\phi}$$

$$Y_3^2 = \frac{1}{2} \sqrt{\frac{105}{32\pi}} \sin^2 \theta \cos \theta e^{2i\phi}; \quad Y_3^3 = \frac{1}{2} \sqrt{\frac{35}{16\pi}} \sin^3 \theta e^{3i\phi}$$

$$(Y_l^m)^* = (-1)^m Y_l^{-m}$$

Note: When calculating terms which are linear in the above coefficients (e.g., interference, polarization, the sign convention becomes important). This table follows the one in Blatt and Weisskopf, *Lectures on Theoretical Physics*, Gamow and Shellery, etc. Other authors (e.g., Schiff, Bethe and de Hoffmann) use different conventions.

QUANTUM MECHANICS II

Do All Problems

1. Using the uncertainty relation calculate the maximum time that a ball bearing (mass = 1 gm) is likely to remain on the top of a perfectly flat table with radius $R = 1$ meter. (Do not consider such real effects as building vibrations and continental drift. Concentrate on the quantum mechanical upper limit.)

2. Consider the helium atom. Estimate the ground state energy either by using perturbation theory or by the variational method.

Potentially useful integrals and formulas.

$$\int d^3r e^{-2kr} = \frac{\pi}{k^3}$$

$$\int d^3r \frac{e^{-2kr}}{r} = \frac{\pi}{k^2}$$

$$\int d^3r_1 d^3r_2 \frac{e^{-2k(r_1 + r_2)}}{|\vec{r}_1 - \vec{r}_2|} = \frac{5}{8} \frac{\pi^2}{k^5}$$

$$\nabla_r^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right)$$

3. a. In atomic physics, the spin orbit interaction has the form $A \vec{l} \cdot \vec{s}$. Derive an expression for the coefficient A in a hydrogenic atom. Don't worry about factors of two, feel free to use qualitative arguments, but get the dependence on the relevant variables correct. Estimate the magnitude of the fine structure splitting in the optical spectrum of sodium.
- b. An electron in a state of orbital angular momentum l is placed in a weak magnetic field. Find an expression for the g-factor.

QUALITATIVE PHYSICS

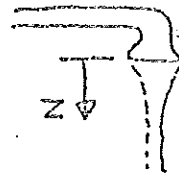
Do all problems.

1. The human body can expel air with a maximum expiratory pressure of about 120 mm Hg. The Indians of the Amazon make use of this pressure to accelerate a poisoned dart in a blowgun.
- Neglecting friction, calculate the muzzle velocity v_m of a 1 gram dart in a 3 meter blowgun of inner cross-sectional area of 1 cm^2 .
 - A resistive force due to the compression of the air ahead of the dart is

$$F_{\text{res}} \approx \rho_{\text{air}} A v^2$$

where $\rho_{\text{air}} = 1.3 \text{ kg/m}^3$. Assume the dart attains a constant velocity state inside the muzzle. How long would you make a blowgun to guarantee that the exit velocity of the dart is within 10% of v_m ?

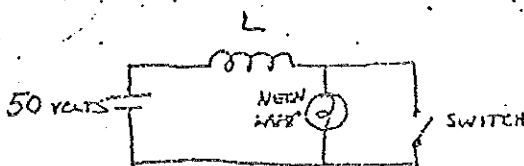
2. A continuous stream of water effluxing downward from the nozzle of a faucet has a characteristic profile.



- Assuming that each element of the fluid is in free fall, find the velocity of the fluid a distance z below the nozzle.
- What is the profile of the water stream as a function of z ? (The profile corresponds to cross-sectional area.)

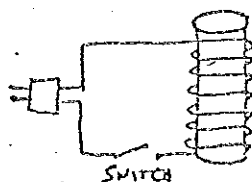
3. POTPOURRI:

- a. Air exhaled from a wide open mouth feels warmer to the back of the hand than air exhaled from a nearly closed mouth even though the volume of air per second emitted is the same. Explain.
 - b. From an atomic viewpoint, why does a blackbody radiate more heat energy than a non-black body which is identical in construction except for color, assuming both are initially at the same temperature?
 - c. A stiff card is placed on top of a drinking glass. A coin is put on the card above the center of the glass and a snap of the first finger off the thumb delivers an appropriate impulsive blow to the edge of the card. What is the condition for the coin to remain on the card?
4. Given the choke circuit with a battery, an inductor, a neon lamp and a switch. Close the switch, then open it. If the neon lamp will not glow below an applied voltage of 90 volts, will the lamp glow at any time during or after the closing or opening of the switch? Explain the physics.



5. An iron bar is suspended from a spring above an electromagnet, their axes coinciding. When a current is passed through the electromagnet, the iron bar is drawn into the core of the electromagnet. (There initially being only an open, non-filled core.)
- a. Give an explanation for the motion of the iron bar.
 - b. Qualitatively explain the subsequent motion of the bar.
 - c. Does Lenz's law apply here? Explain.

6. A repulsion coil constructed from 1000 turns of #12 insulated copper wire around a tube (2 cm diameter) is plugged into a 120 volt outlet. The 10 amp fuse blows up.
- By substituting a 10 amp slow blow fuse, the operation of the coil proceeds uninterrupted. Why?
 - Add iron rods (these rods are long enough to protrude out the top) down the core center. Will the current drawn at 120 volts increase or decrease? Explain.
 - An aluminum ring placed around the coil just above the center jumps upward when the coil is being energized. Explain the physics.
 - A nonconducting ring does not jump during energization. Will it have an EMF around it when placed in the changing field?



7. An optical diffraction grating is subject to distortions resulting from the lack of care and quality control. Qualitatively, in terms of the resolution, linewidth and angular dispersion, explain the features of the diffraction pattern introduced by
- a non-planar grating
 - unequal slit spacings
 - unequal slit widths
 - tilt with respect to the incident beam
 - non-uniform slit widths (trapezoiding).

8. Estimate time in seconds for:

- a. age of universe
- b. lifetime of excited electron in an atom
- c. collision time of an air molecule
- d. time of flight of light from sun to earth
- e. decay time of LCR (inductance, capacitance, resistance) circuit with
 $L = 1 \text{ Henry}$, $C = 1 \mu\text{F}$, $R = 20 \Omega$
- f. average half-life of C^{14} for radiocarbon dating
- g. sound to travel down a 100 meter well and echo back.

9. A recent safety rope invention in mountaineering is an elastic rope which stretches by an amount ΔL proportional to the applied tension T :

$$T = \gamma \frac{\Delta L}{L_0}$$

L_0 being the length at zero tension. Assume that a climber falls from a height L_0 above the last piton where the rope is anchored.

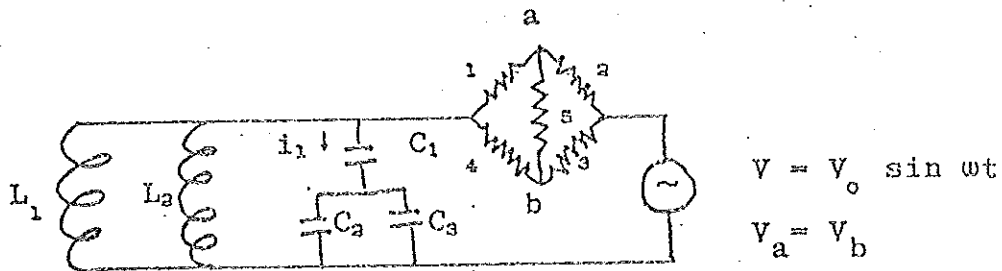
- a. Find the velocity v_0 after he falls a distance $2L_0$ and choose this point as the zero of time and where $z = 0$.
 - b. Qualitatively graph the motion of the falling body.
 - c. Given $\gamma = 10^4$ lbs. and a 150 lb. climber, what is the maximum tension T and show that this maximum T is independent of the height of the fall, $2L_0$.
10. a. Two church bells which appear to be identical are struck by hammers imparting the same impulse. However, they emit a different quality of sound, the better one resonating for twice as long as the other. Explain the differences in their sound characteristics in physics terms such as frequency spectrum, the Q , and the decay time.
- b. Organ pipes also exhibit sound resonance. Independent of the excitation method, organ pipes produce harmonics. Using Fourier analysis, qualitatively discuss the harmonics expected from open and closed (at one end) pipes.

ELECTRICITY AND MAGNETISM

Do 4 Problems (Closed Book)

1. A parallel plate capacitor of area A , plate separation d , and dielectric ϵ is charged and then disconnected from the charging source.
- Calculate \vec{E} , \vec{D} , the potential difference between the plates, and the capacitance.
 - What force is required to keep the plates apart?
 - How much work is required to remove the dielectric?, to completely disassemble the capacitor?
 - Under what condition is the fringing field negligible?

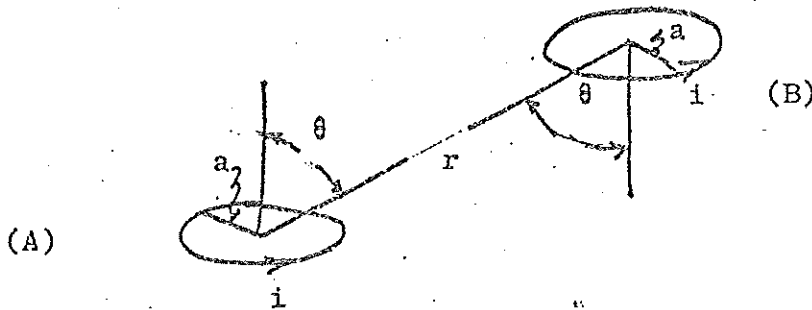
2.



Write differential equation for the currents i , in the above circuit. Solve for steady steady condition.

3. Calculate:

- the force between the two current carrying loops shown in the figure.
- the torque exerted by one loop on the other.



$$\frac{a}{r} \ll 1$$

4. Write the equation for the propagation of a plane electromagnetic wave in a conducting medium.

- Assuming a monochromatic wave, how far will it penetrate into the medium?
- Is the field transverse or longitudinal?
Discuss your answer.

5. a. What is the force per unit length between two very long parallel current-carrying conductors?

Apply this result to explain the self-compression of a current-carrying neutral plasma (pinch effect).

- Consider two electrons moving parallel to each other at a velocity v in the laboratory.

What is the force between the electrons for $v = 0$, $v = c$?

